MATHEMATICS

Mob. : 9470844028
9546359990

AIM POINT
MATHEMATICS
DIR. FIROZ AHMAD
M.Sc. (Maths), B.Ed, M.Phil (Maths)

RAM RAJYA MORE, SIWAN

XIth, XIIth, TARGET IIT-JEE
(MAIN + ADVANCE) & COMPATETIVE EXAM
FOR XII (PQRS)

AREA BOUNDED BY CURVES

& Their Properties

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THINGS TO REMEMBER

✷ Curve Tracing

For the evaluation of area of bounded regions it is very essential to know the rough sketch of the curves. The following points are very use full to draw a rough sketch of a curve.

Symmetry

(i) Symmetry about *x*-axis If the equation of the curve remains unaltered when y is replaced by –y, then the curve is symmetrical about x-axis.

(ii) Symmetry about *y*-axis If the equation of the curve remains unaltered when x is replaced by $-x$, then the curve is symmetrical about y-axis.

(iii) Symmetry about $y = x$ If the equation of the curve remains unaltered if x and y are interchanged, then the curve is symmetrical about $y = x$.

(iv) Symmetry about $y = -x$ If x and y are replaced by –y and $-x$ and the equation of the curve remains unaltered, then the curve is symmetrical about $y = -x$.

(v) Symmetry in Opposite Quadrants If the equation of the curve remains unaltered when x and y replaed by –x and –y, then it is symmetrical about $y = x$.

Intersection with Origin

If the constant term in the equation of curve is zero, then curve passes through the origin.

Intersection with Coordinate Axes

(i) For finding intersection points of the curve with the x-axis put y = 0 and solve equation for x. Toots of equation gives points of intersection with x-axis.

(ii) Similarly for finding intersection points with the y-axis put $x = 0$ and solve the equation for y. Roots of equation give points of intersection with y-axis.

Asymptotes

If asymptotes are parallel to x-axis or parallel to y-axis, then to find asymptotes

(i) Find y in terms of x and, if there exist real value(s) of x such that the denominator becomes zero, then x equals to that value(s), is(are) asymptote(s) to that curve.

(ii) Similarly find x is terms of y and, if there exist real value(s) of y such that the denominator be comes zero, then y equals to that value(s), is(are) asymptote(s) to that curve.

eg, Let the equation of the curve be $x^2y^2 = a^2(y^2 - x^2)$ then

$$y^2 = \frac{-a^2x^2}{x^2 - a^2}$$

$x^2 - a^2$ becomes zero, if $x = \pm a$

$\therefore\ x = \pm a$ are the asymptotes parallel to y-axis.

Tangents at the Origin

See whether the curve passes though origin or not. If it passes through the origin then to find the equation of tangents at the origin, equate the lowest degree term to zero.

eg, $y^3 = x$ passes through the origin. The lowest degree term in the equation is x, then on equating x to zero , we get $x = 0$

So, $x = 0$ ie, y-axis is tangent at the origin to $y^3 = x$.

Regions where the Curve does not Exist

For this, find the value of y in term of x from the equation of the curve and find the values of x for which y is imaginary. Similary, find the value of x in terms of y and determine the value of y for which x is imaginary. The curve does not exist for these values of x and y.

✱ Area of Curve Given by Cartesian Equations

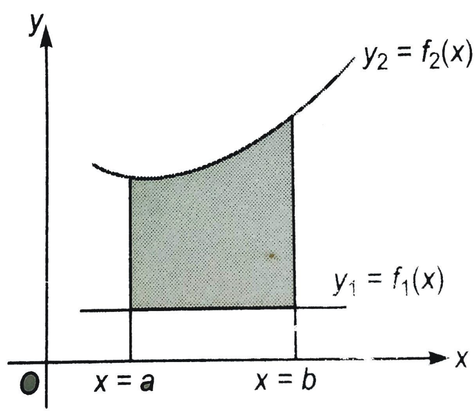
Let $y = f(x)$ be a finite and continuous function in the interval [a, b]. Then, the area between the curve $y = f(x)$, x-axis and two ordinates at the points $x = a$ and $x = b$ is given by the formula.

$$A = \int_a^b f(x)dx = \int_a^b ydx$$

Similarly, the area between the curve $x = g(y)$, y-axis and two abscissage $y = c$ and $y = d$ is $\int_c^d g(y)dy = \int_c^d xdy$.

Area Bounded between Two Curves

Let $y_1 = f_1(x)$ and $y_2 = f_2(x)$ be two curves. Then the area (along x-axis) bounded between the curves and the lines $x = a$ and $x = b$ is given by



$$A = \int_a^b \{f_2(x) - f_1(x)\}dx$$

$(f_2(x) > f_1(x)$ in [a, b])

$$= \int_a^b \{y_2 - y_1\}dx$$

Similarly, Area (along y-axis) bounded by the curves $x_1 = f_1$ (y) and $x_2 = f_2(y)$ between $y = c$ and $y = d$ is given by

$$A = \int_c^d \{f_2(y) - f_1(y)\}dy$$

$$= \int_c^d \{x_2 - x_1\}dy$$

Note :

- If function is periodic and we can find it's period, then plot curve for the interval equal to one period and repeat it.
- If $f(a)\, f(b) < 0$, curve intersects x-axis at least once. Similarly, if $f(a)\, f(b) > 0$ curve intersects x-axis even number of times.
- Areas under the x-axis will come out negative and areas above the x-axis will be positive. This mean you have to be careful when finding and area which is partly above and partly below tha x-axis.